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13. ABSTRACT (Maximum 200 words)  We have developed a general approach to the problem of characterizing generic classes of objects and recognizing them in images. We consider only objects composed of simple parts, where the classes are defined by constraints on the properties of and relations among the parts. Object classes are characterized ("modeled") in terms of both qualitative part properties and relations.					
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# FINAL REPORT

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## Technical Report Abstracts

1. Chung-Nim Lee and Azriel Rosenfeld, "Computing the Euler Number of a 3D Image." CAR-TR-205, CS-TR-1667, May 1986.

**ABSTRACT:** A new efficient algorithm for computing the Euler number of a 3D digital image  $S$  is given. It is based on deep mathematical principles in differential geometry and algebraic topology. It runs in  $O(n)$  time where  $n$  is the number of object elements of the border image  $Bd\ S$ . The basic idea is so general that it can be easily generalized to images defined by other data structures such as a hierarchical data structure or a union of isothetic (hyper-)rectangles.

2. Azriel Rosenfeld, "A Note on Average Distances in Digital Sets." CAR-TR-206, CS-TR-1670, June 1986.

**ABSTRACT:** For any compact connected metric space  $S$ , there exists a unique nonnegative real number  $\bar{d}$  such that, for any positive integer  $n$  and any points  $P_1, \dots, P_n$  of  $S$ , there exists a point  $P$  in  $S$  whose average distance from the  $P_i$ 's is exactly  $\bar{d}$ . In this note we prove that for any finite connected digital set  $S$  and any integer-valued metric defined on  $S$ , there exists a nonnegative integer  $\bar{d}$  such that, for any positive integer  $n$  and any points  $P_1, \dots, P_n$  of  $S$ , there exists a point  $P$  in  $S$  whose average distance from the  $P_i$ 's differs from  $\bar{d}$  by at most  $1/2$ ; but  $\bar{d}$  is not necessarily unique.

3. Chung-Nim Lee and Azriel Rosenfeld, "Simple Connectivity is Not Locally Computable for Connected 3D Images." CAR-TR-207, CS-TR-1671, June 1986.

**ABSTRACT:** It is commonly known that the connectivity predicate "is connected" is not locally computable for 2D digital images, and hence for any dimensional images. In this note, we show that the predicate "is simply connected" is locally computable for connected 2D images, but not for connected 3D images. We also show that orientability of a surface is not locally computable.

4. Michael Werman, Sam Baugher and J. Anthony Gualtieri, "The Visual Potential: One Convex Polygon." CAR-TR-212, CS-TR-1690, August 1986.

**ABSTRACT:** We examine the visual potential of a single convex polygon with  $n$  faces. The visual potential (also called the aspect graph) is a mathematical representation of the stable views of a polygon and the abrupt changes in stable views brought about by moving the viewpoint. Stable views, *aspects*, are described by the *topological* structure—incidence relations—of the visible vertices and faces which are projected onto the retina of a cyclopean eye. While the *metrical* structure may vary, the topological structure is invariant to most small changes in viewpoint. Abrupt changes in aspect, *visual events*, correspond to the appearance and disappearance of vertices and faces. For a convex polygon visual events occur when the viewpoint crosses any line containing a face of the

polygon. These lines partition the space around the object into regions. All viewpoints within a region see the same aspect. Aspects and visual events correspond respectively to the nodes and edges of a graph that represents the visual potential. A given visual potential describes a class of object instances which have the same *shape* or surface topology. When metrical attributes such as scale, length, and angle are added to the class description a particular instance of the object class is realized. We examine the local global structure of the visual potential and prove that a *cylindrical chain link fence* can serve as its underlying manifold. In addition we obtain an optimal algorithm,  $O(n)$ , for finding the *maximal aspects*, a set of aspects which encode the entire visual potential, and an  $O(n^2)$  algorithm for generating the entire visual potential from these maximal aspects. We also give bounds on the number of *isomorphism classes* (i.e. number of shapes) for convex polygons with a fixed number of faces. We discuss realizability of visual potentials, relationship to other representations, extension to three dimensions, and an extension to a class of non-convex polygons.

5. Angela Y. Wu, S.K. Bhaskar and Azriel Rosenfeld, "Parallel Computation of Geometric Properties from the Medial Axis Transform." CAR-TR-213, CS-TR-1691, August 1986.

ABSTRACT: The Medial Axis Transform (MAT) represents a region of a digital image as the union of maximal upright squares contained in the region. This paper presents algorithms to compute the contour, perimeter and area of the region covered by a set of  $n$  upright rectangles in  $O(n)$  time using  $O(n)$  processors for the shared memory model, and using  $O(n^2)$  processors for mesh-connected computers.

6. Millu Rosenblatt-Roth, "Contributions to a General Theory of Markov Meshes: I. Definition, Existence Theorems, Cardinality of Characteristic Sets." CAR-TR-215, CS-TR-1693, November 1986.

ABSTRACT: This paper introduces the concept of a Markov mesh as a subclass of the class of multidimensional Markov fields, which possesses all the important properties suggested by Markov chains. Existence theorems are proved and the cardinality of characteristic sets is studied. The sets of states are arbitrary and no homogeneity is assumed.

7. Millu Rosenblatt-Roth, "Contributions to a General Theory of Markov Meshes: II. Homogeneity in Markov Meshes and Markov Fields." CAR-TR-216, CS-TR-1694, November 1986.

ABSTRACT: This paper introduces the concept of homogeneity and complex homogeneity in Markov random fields and in Markov meshes. It studies the characteristic sets in this case.

8. Millu Rosenblatt-Roth, "Contributions to a General Theory of Markov Meshes: III. The Temporal Representation of Spatial Markov Meshes." CAR-TR-217, CS-TR-1695, November 1986.

ABSTRACT: This paper contains the proof that each multidimensional Markov mesh has at least one isomorphic probability preserving representation as a temporal (one dimensional) Markov mesh, and that this property is characteristic for a Markov mesh.

9. Sharat Chandran and Azriel Rosenfeld, "Order Statistics on a Hypercube." CAR-TR-228, CS-TR-1719, October 1986.

ABSTRACT: We consider an order statistics problem, a variant of the selection problem. We present an optimal parallel algorithm to find the highest  $k$  elements out of a set of  $n$  totally ordered (but not sorted) elements. This algorithm runs on a hypercube multiprocessor in  $O(n/p)$  time ( $p$  is the number of processors) using up to  $O(n^{1-\theta})$  processors,  $0 < \theta < 1$ .

10. T. Yung Kong and Azriel Rosenfeld, "Digital Topology: Introduction and Survey." CAR-TR-229, CS-TR-1720, October 1986.

ABSTRACT: Digital topology deals with the topological properties of digital images; or, more generally, of discrete arrays in two or more dimensions. It provides the theoretical foundations for important image-processing operations such as connected component labeling and counting, border following, and thinning, and their generalizations to three- (or higher-) dimensional "images". This paper reviews the fundamental concepts of digital topology, and surveys the major theoretical results in the field. A bibliography of over 80 references is included.

11. Kamala Krithivasan and R. Sitalakshmi, "An Efficient Two Dimensional Pattern Matching Algorithm." CAR-TR-233, CS-TR-1724, October 1986.

ABSTRACT: In this paper we consider an efficient two dimensional pattern matching algorithm. R.S. Bird [2] has given an algorithm of complexity  $O(m_1 m_2 + n_1 n_2)$  where the pattern is of size  $n_1 \times n_2$  and the text is of size  $m_1 \times m_2$ . This uses the KMP algorithm [4] along both rows and columns. Since the BM algorithm [3] is known to perform better in many cases, we use BM along the rows and KMP along the columns. The worst case complexity is  $O(n_1^2 n_2 + m_1 m_2 + \# \Sigma)$  when we use both delta 1 and delta 2 tables of the BM algorithm, and  $O(n_1 n_2 + m_1 m_2 + \# \Sigma)$  if we use just the delta 1 table of BM. In any case empirical evidence shows that our algorithm performs better than Bird's algorithm, when the alphabet size is large.

12. Kamala Krithivasan, "Time Varying Pushdown Automata." CAR-TR-242, CS-TR-1741, November 1986.

**ABSTRACT:** Time varying pushdown automata (PDA) are defined and equivalence between two modes of acceptance shown. It is seen that periodically time varying pushdown automata accept exactly the class of context-free languages. Time varying generalized PDA are defined and their equivalence to terminal weighted context free grammars in GNF shown. It is shown that TVGPDA can be simulated by TVPDA. Thus TVPDA give another machine characterization of recursively enumerable sets.

13. Subir Kumar Ghosh, "Computing the Visibility Polygon from a Convex Set." CAR-TR-246, CS-TR-1749, December 1986.

**ABSTRACT:** In this paper, we propose efficient algorithms for computing the complete and weak visibility polygons of a simple polygon  $P$  of  $n$  vertices from a convex set  $C$  inside  $P$ . The algorithm for computing the complete visibility polygon of  $P$  from  $C$  takes  $O(n+k)$  time in the worst case where  $k$  is the number of straight line segments enclosing the convex set  $C$ . Given a triangulation of  $P-C$ , the algorithm for computing the weak visibility polygon of  $P$  from  $C$  takes  $O(n+k)$  time in the worst case. We also show that the complete and weak visibility polygons of  $P$  from a nonconvex set inside  $P$  can be computed in the same time complexity.

14. Millu Rosenblatt-Roth, "Contributions to the Problem of Approximation of Probability Measures. I. The Problem." CAR-TR-249, CS-TR-1754, January 1987.

**ABSTRACT:** This is the first in a series of Technical Reports dedicated to the presentation of basic aspects concerning the approximation of probability measures by simpler ones. It discusses a program of modeling by approximation and gives some of the results obtained by the author in this direction.

15. Millu Rosenblatt-Roth, "Contributions to the Problem of Approximation of Probability Measures. II. On the Meaning of Shannon's Amount of Information." CAR-TR-250, CS-TR-1755, January 1987.

**ABSTRACT:** This Technical Report contains the solution of the problem of best approximation of an arbitrary given pair of random variables with the help of a pair of independent random variables. We determine explicitly the best approximants and prove that the corresponding least error is Shannon's amount of information. The solution of this problem is a model for more complex situations.

16. Millu Rosenblatt-Roth, "Contributions to the Problem of Approximation of Probability Measures. III. On the Best Approximation of Probability Measures." CAR-TR-251, CS-TR-1756, January 1987.

ABSTRACT: We find the best approximation of a probability measure with probability measures of various classes; the least errors are information functionals, the simplest of which is Shannon's "amount of information."

17. Chung-Nim Lee and Azriel Rosenfeld, "Representation of Orthogonal Regions by Vertices." CAR-TR-257, CS-TR-1766, January 1987.

ABSTRACT: Denote by  $X$  a finite union of orthogonal regions in  $n$  dimensions ( $n=2, 3$ ), and by  $\partial X$  its boundary polyhedron, in which self-intersections are allowed. Certain vertices of  $\partial X$  are referred to as corner vertices and are further divided into two mutually exclusive types depending on the number of incident edges (for  $n=2$ ) and faces (for  $n=3$ ). We present simple algorithms for recovering the region  $X$  and the polyhedron  $\partial X$  from the set of corner vertices together with specification of their types.

18. Kamala Krithivasan and R. Sitalakshmi, "Efficient Two Dimensional Pattern Matching in the Presence of Errors." CAR-TR-259, CS-TR-1768, January 1987.

ABSTRACT: We give an algorithm for two dimensional pattern matching in the presence of errors. We find that the complexity of our algorithm is  $O(k n_1 n_2 \log n_2 + n_1^2 n_2 + k n_1 m_1 m_2)$  where the pattern is an  $n_1 \times n_2$  array, the text is an  $m_1 \times m_2$  array and  $k$  is the number of mismatches allowed.

19. Kamala Krithivasan, "Efficient Two Dimensional Parallel and Serial Approximate Pattern Matching." CAR-TR-266, CS-TR-1786, February 1987.

ABSTRACT: We consider the two-dimensional pattern matching problem in which the pattern is an  $m_1 \times m_2$  rectangular array and the text is an  $n_1 \times n_2$  array, and differences between characters of the pattern and characters of the text are allowed. Each difference is due to either a mismatch between a character of the text and a character of the pattern or a superfluous character in the text or a superfluous character in the pattern. We present parallel and serial algorithms for finding all occurrences of the pattern in the text with at most  $k$  differences. The parallel algorithm consists of an analysis of the pattern and takes  $O(\log m_2)$  time on  $m_1 m_2^2$  processors. The other two parts handle the text.

The second part takes  $O(\log m_2)$  time on  $\frac{m_1 n_2}{\log m_2}$  processors and the third part takes  $O(n_1 k)$  time on  $m_1(n_2 + k + 1)$  processors. In the case of the serial algorithm, the running time is  $O(k m_1 n_1 n_2)$  for an alphabet whose size is fixed and  $O(n_2(k + \log m_2)n_1 m_1)$  for general input. In both cases the space requirement is  $O(m_1 n_2)$ . In the case of the serial algorithm we can effectively reduce the time

if some rows of the pattern are identical by doing a preprocessing of the pattern.

20. Kamala Krithivasan and Paliath Navendran, "On the Membership Problem for Some Grammars." CAR-TR-267, CS-TR-1787, February 1987.

ABSTRACT: We prove that the membership problem for growing CSG is NP-complete if  $G$  is given as input, thus answering an open problem posed by Dahlhaus and Warmuth. We also show the membership problem for EMG, SNG, IPG, DVCFGG, and FRCFGG to be NP-complete if  $G$  is given as input. If  $G$  is fixed, the membership problem for EMG is shown to be in  $P$ .

21. Chengye Wang, Liuqing Huang and Azriel Rosenfeld, "Detecting Clouds and Cloud Shadows on Aerial Photographs." CAR-TR-268, CS-TR-1788, February 1987.

ABSTRACT: This paper describes a system for identifying small clouds and their shadows on aerial photographs. The system segments an image into homogeneous regions; selects bright and dark regions as cloud and shadow candidates, respectively; and finds acceptable (cloud, shadow) pairs based on consistent relative position. The system performed quite well on three portions of aerial photographs each of which contained 10-20 small clouds (out of 25-60 bright regions). Other clues could have been used to aid in cloud identification; for example, clouds have irregular shapes, do not have long straight edges, and may occlude edges or curves on the terrain. However, it was not found necessary to use such clues in our examples.

22. Subir Kumar Ghosh, "A Few Applications of the Set-Visibility Algorithm." CAR-TR-273, CS-TR-1797, March 1987.

ABSTRACT: In this paper, we propose efficient algorithms for the following problems in computational geometry. (i) Given a polygon  $Q$  of  $m$  vertices inside another polygon  $P$  of  $n$  vertices, the problem is to construct a convex polygon  $C$  nested between  $P$  and  $Q$  with the minimum number of vertices. The algorithm runs in  $O((n+m)\log k)$  time where  $k$  is the number of vertices of  $C$ . (ii) Given two points inside a polygon  $P$ , the problem is to compute a path between them inside  $P$  with the minimum number of line segments. Given a triangulation of  $P$ , the algorithm takes  $O(n)$  time. Both the algorithms are based on the algorithm in [G86] for computing the *complete* visibility polygon of  $P$  from a set inside  $P$ .

23. Angela Y. Wu, S.K. Bhaskar and Azriel Rosenfeld, "Parallel Processing of Regions Represented by Linear Quadrees." CAR-TR-296, CS-TR-1863, June 1987.

ABSTRACT: We show how computation of geometric properties of a region represented by a linear quadtree can be speeded up by about a factor of  $p$  by using a  $p$ -processor CREW PRAM model of parallel computation. Similar

speedups are obtained for computing the union and intersection of two regions, and the complement of a region, using linear quadtree representations.

24. Subir Kumar Ghosh and David M. Mount, "An Output Sensitive Algorithm for Computing Visibility Graphs." CAR-TR-302, CS-TR-1874, July 1987.

ABSTRACT: The visibility graph of a set of nonintersecting polygonal obstacles in the plane is an undirected graph whose vertices are the vertices of the obstacles and whose edges are pairs of vertices  $(u,v)$  such that the open line segment between  $u$  and  $v$  does not intersect any of the obstacles. The visibility graph is an important combinatorial structure in computational geometry and is used in applications such as solving visibility problems and computing shortest paths. An algorithm is presented that computes the visibility graph of a set of obstacles in time  $O(E + n \log n)$ , where  $E$  is the number of edges in the visibility graph and  $n$  is the total number of vertices in all the obstacles.

25. Reinhard Klette and Klaus Voss, "The Three Basic Formulae of Oriented Graphs." CAR-TR-305, CS-TR-1883, July 1987.

ABSTRACT: As a theoretical foundation for processing of digital pictures a theory of *oriented graphs* is presented. The orientation of a graph is defined by cyclic enumerations of all of its star subgraphs. For these oriented graphs the notion of a *mesh* is of central importance, characterizing a complete period of an oriented path of such a graph. It will be shown that the topological theory of oriented graphs may be based upon the Vertex Formula, the Mesh Formula, and the Euler Formula. Neighborhood structures as used in image processing are oriented graphs, and the topological meaning of these three formulae will be demonstrated by deducing theoretical facts and methods which are of direct practical relevance to digital image processing. The computation and analysis of border meshes is dealt with, formulae of Pick type are proved, and the theory is applied to homogeneous as well as non-homogeneous structures.

26. David M. Mount and Ruth Silverman, "Packing and Covering the Plane with Translates of a Convex Polygon." CAR-TR-324, CS-TR-1917, October 1987.

ABSTRACT: A covering of the Euclidean plane by a polygon  $P$  is a system of translated copies of  $P$  whose union is the plane, and a packing of  $P$  in the plane is a system of translated copies of  $P$  whose interiors are disjoint. A lattice covering is a covering in which the translates are defined by the points of a lattice, and a lattice packing is defined similarly. We show that, given a convex polygon  $P$  with  $n$  vertices, the densest lattice packing of  $P$  in the plane can be found in  $O(n)$  time. We also show that the sparsest lattice covering of the plane by a centrally symmetric convex polygon can be solved in  $O(n)$  time. Our approach utilizes results from classical geometry that reduce these packing and covering problems to the problems of finding certain extremal enclosed figures within the polygon.

27. Azriel Rosenfeld, "Coordinate Grammars Revisited: Generalized Isometric Grammars." CAR-TR-325, CS-TR-1918, September 1987.

ABSTRACT: In a "coordinate grammar", the rewriting rules replace sets of symbols having given coordinates by sets of symbols whose coordinates are given functions of the coordinates of the original symbols. It was shown in [1] that coordinate grammars are "too powerful"; even if the rules are all of finite-state types, and the functions are all computable by finite transducers, the grammar has the power of a Turing machine. This paper shows that if we require the functions to be shift-invariant and the rules to be of bounded diameter, then such grammars do have a useful hierarchy of types; in fact, when we require that their sentential forms always remain connected, they turn out to be equivalent to "isometric grammars" [2].

28. Avraham Margalit and Gary D. Knott, "An Algorithm for Computing the Union, Intersection or Difference of Two Polygons." CAR-TR-350, CS-TR-1995, March 1988.

ABSTRACT: A robust algorithm for set operations on pairs of polygons is presented. The algorithm is capable of operating on the class of vertex-complete polygons which properly includes the simple polygons. The algorithm uses carefully chosen data structures and is easy to describe. We present the algorithm and its implementation and give a proof of its correctness and an analysis of its complexity.

29. S.K. Bhaskar, Azriel Rosenfeld and Angela Wu, "Models for Neighbor Dependency in Planar Dot Patterns." CAR-TR-351, CS-TR-2000, March 1988.

ABSTRACT: Random processes are defined that generate planar dot patterns in which the dots have a tendency to cluster, or in which clustering is inhibited. Processes are also defined for labeling a given point pattern in such a way that neighboring points tend to have, or not to have, the same labels.

30. Azriel Rosenfeld, "Explaining Noisy Data: A Qualitative Bayesian Approach." CAR-TR-358, CS-TR-2022, April 1988.

ABSTRACT: When we examine a set of data, it is often "obvious" that the data can be interpreted as values of a particular type of function (e.g., linear) corrupted by a particular type of noise (e.g., zero-mean, spatially stationary). This paper presents a qualitative approach, based on Bayes' theorem, that may justify such interpretations. The discussion deals primarily with data that are samples of a real-valued function of a single variable, but we also indicate how similar ideas apply to functions of two or more variables, to vector-valued functions (e.g., curves or surfaces), as well as to the problem of finding natural clusters in sets of points.

31. Azriel Rosenfeld and Angela Y. Wu, ""Digital Geometry" on Graphs." CAR-TR-371, CS-TR-2065, July 1988.

**ABSTRACT:** Many of the standard concepts of digital geometry, particularly those involving connectedness and distance properties of subsets of a digital image, can be generalized to subgraphs of an arbitrary graph  $G$ . Algorithms for connected component labeling, distance transform computation, etc. can be defined that require time  $O(n)$ , where  $n$  is the number of nodes of  $G$ . Parallel algorithms for these computations can also be defined, using various modes of parallel computation. We can also define "continuous" integer-valued functions on graphs, and can show that the distance transform is the largest such function having (at least) a given set of zeros.

32. Prabir Bhattacharya and Azriel Rosenfeld, "Contour Codes of Isothetic Polygons." CAR-TR-382, CS-TR-2092, August 1988.

**ABSTRACT:** An isothetic polygonal arc is one that has all its sides oriented in two orthogonal directions, so that all its angles are right angles. Such an arc is determined (up to congruence) by specifying a "code" sequence of the form  $\alpha_1 A_1 \alpha_2 \cdots \alpha_{m-1} A_{m-1} \alpha_m$ , where the  $\alpha$ 's are positive real numbers representing side lengths, and the  $A$ 's are single bits that specify whether the arc turns left or right between one side and the next. In this paper we develop basic properties of this code, and show how to derive various geometric properties of the arc (or the region it bounds, if it is closed) directly from the code.

33. Avraham Margalit and Azriel Rosenfeld, "Using Feature Probabilities to Reduce the Expected Computational Cost of Template Matching." CAR-TR-386, CS-TR-2097, August 1988.

**ABSTRACT:** Matching of two digital images is computationally expensive, because it requires a pixel-by-pixel comparison of the pixels in the image and in the template for every location in the image. In this paper we present a technique to reduce the computational cost of template matching by using probabilistic knowledge about local features that appear in the image and the template. Using this technique the most probable locations for successful matching can be found. In the paper we discuss how the size of the features affects the computational cost and the robustness of the technique. We also present results of experiments showing that even simple methods of feature extraction and representation can reduce the computational cost by more than an order of magnitude.

34. Avraham Margalit and Azriel Rosenfeld, "Reducing the Expected Computational Cost of Template Matching Using Run Length Representation." CAR-TR-406, CS-TR-2143, November 1988.

**ABSTRACT:** Template matching of two digital images, represented as arrays of pixels, is computationally expensive, because it requires a pixel-by-pixel comparison of the pixels in the image and in the template for every location in the

image. In this paper we present an algorithm to reduce the computational cost of template matching by using run length representation of the image and the template. Using this technique we compare only locations in the image and the template where the total mismatch accumulation may change. This method works best for images and templates with long runs. In the paper we present the algorithm, discuss conditions for it being efficient, and show experimental results on both randomly generated and real images. We present some results in which using this method yields more than 20-fold speedup.

35. Sharat Chandran and David Mount, "Optimal Shared Memory Parallel Algorithms and the Medial Axis Transform." CAR-TR-411, CS-TR-2165, December 1988.

**ABSTRACT:** In computer vision, the Medial Axis Transform represents a region of a digital image as the union of maximal upright squares contained in the region. In this paper we consider the problem of computing geometric properties of the image from a representation that generalizes the squares to rectangles. We give algorithms for a number of problems using  $n$  processors where  $n$  is the number of upright rectangles. Our algorithms compute the perimeter, eccentricity, center of gravity, moment of inertia and area of the region covered by the rectangles in  $O(\log n)$  time. Our result is faster than previous results and is optimal (to within a constant factor). The *contour* of such a region may contain as much as  $O(n^2)$  pieces; our algorithm computes the contour with a worst case running time of  $O(n)$ .

Since a two dimensional array is the standard method by which sensors usually provide the image, it is of interest to compute the MAT from an array representation. We give an optimal parallel algorithm to construct the medial axis transform representation given an array representation of the image.

36. Avraham Margalit and Azriel Rosenfeld, "Matching Polygonal Arcs." CAR-TR-418, CS-TR-2174, January 1989.

**ABSTRACT:** In this paper we present an efficient algorithm for matching two rectilinear polygonal arcs. We first show how to match two arcs of the same length by decomposing them into a set of pairs of corresponding straight line segments having the same length. The distance measure of each such pair of line segments is calculated by referring to the distance of one of six possible configurations of pairs of segments. Then we show how to find the relative position of the two arcs which yields the best match by minimizing the distance function. After analyzing the case of arcs having the same length, we show how to use the results and a representation of rectilinear arcs as strings generated by four primitives to get an efficient algorithm for arc matching. This algorithm is based on an algorithm for run-length string matching we presented in a previous paper.

37. Avraham Margalit and Gary Knott, "An Algorithm for Computing the Union, Intersection or Difference of Two Sets of Polygons." CAR-TR-419, CS-TR-2175, January 1989.

**ABSTRACT:** An algorithm for set operations on two sets of polygons is presented. The algorithm uses a boundary representation for the input and output polygon sets. The polygons in each set can be either island or hole polygons. They can be simple polygons as well as vertex-complete polygons with dangling edges, vertices of degree greater than 2. A hole within the area of a polygon may be specified by another polygon specified to be a "hole" polygon. We give the basic algorithm and show a way to make it more efficient by using inclusion trees that can be built from the polygons in each set. A partial proof of the correctness of the algorithm is given as well as a basic analysis of its complexity. The suggested implementation is table-driven and is facilitated by the use of efficient data structures. Sample results of an implementation of the algorithm are demonstrated.

38. Behzad Kamgar-Parsi, Avraham Margalit and Azriel Rosenfeld, "Matching General Polygonal Arcs." CAR-TR-442, CS-TR-2247, May 1989.

**ABSTRACT:** In this paper we present an algorithm for matching two polygonal arcs. We first show how to match two arcs of the same length by decomposition into pairs of equal length line segments. The distance measure used is the sum of the squared distances between corresponding points on the two arcs. We find the best match by calculating analytically the relative position and orientation of the arcs that minimize the distance measure. After analyzing the case of arcs having the same length, we show how to use this result in the algorithm for arc matching, illustrate its application with an example, and suggest how to implement it efficiently. We also discuss simplifications of the algorithm in the case of digital images.

39. Sven J. Dickinson, Alex P. Pentland and Azriel Rosenfeld, "A Representation for Qualitative 3-D Object Recognition Integrating Object-Centered and Viewer-Centered Models." CAR-TR-453, CS-TR-2272, June 1989.

**ABSTRACT:** Two important issues arise in the representation of objects for 3-D object recognition. The first issue is the choice between object-centered and viewer-centered representations. Object-centered representations model objects as constructions of 3-D primitives, such as planar faces or generalized cylinders. Viewer-centered representations model objects as a set of 2-D characteristic views, or aspects. The advantage of viewer-centered representation is that it reduces 3-D recognition to 2-D recognition; solving the inverse projection problem is unnecessary. However, with each model object having potentially many aspects, matching becomes less efficient than with object-centered models. The second issue concerns the amount of detail inherent in object models. Quantitative models facilitate simple, model-based verification procedures at the expense of model complexity. Qualitative models preclude top-down verification, but are

invariant to minor changes in shape.

This paper proposes a modeling paradigm for 3-D object recognition integrating object-centered and viewer-centered models. Object models are object-centered constructions of 3-D volumetric primitives, offering an efficient indexing mechanism for large object databases. The 3-D primitives, in turn, are mapped into a set of viewer-centered aspects. To minimize the size of the aspect set, we constrain the aspects to be invariant to minor changes in primitive shape, forcing the primitives to be qualitative in nature. Primitive reconstruction matches local 2-D image features to the set of viewer-centered aspects, whose size depends only on the size of the set of primitives, not on the number of object models or on object model complexity. Object recognition then matches the primitives to object-centered models. To accommodate incomplete aspects arising from occluded model primitives, we introduce a hierarchical aspect representation based on aspect faces. The levels of the hierarchy are linked together by a set of conditional probabilities resulting from an extensive analysis of the aspects.

40. Annick Montanvert, Peter Meer and Azriel Rosenfeld, "Hierarchical Image Analysis Using Irregular Tessellations." CAR-TR-464, CS-TR-2322, September 1989.

**ABSTRACT:** We present a novel multiresolution image analysis technique based on hierarchies of irregular tessellations generated in parallel by independent stochastic processes. Like the "traditional" image pyramids these hierarchies are constructed in on the order of  $\log(\text{image\_size})$  steps. However, the structure of a hierarchy is adapted to the image content and artifacts of rigid resolution reduction are avoided. We give two applications of our technique: connected component analysis of labeled images, and segmentation of gray level images. In labeled images, every connected component is reduced to a separate root, with the adjacency relations among the components also extracted. In gray level images the output is a segmentation of the image into a small number of classes as well as the adjacency graph of the classes.

41. Azriel Rosenfeld, T. Yung Kong and Angela Y. Wu, "Digital Surfaces." CAR-TR-467, CS-TR-2329, October 1989.

**ABSTRACT:** A three-dimensional digital object is a union of voxels, i.e., upright unit cubes whose vertices have integer coordinates. The surface of such an object thus consists of surface pixels, i.e., unit squares parallel to the coordinate planes and whose vertices have integer coordinates. This paper discusses some basic properties of digital surfaces.

42. Azriel Rosenfeld, "Image Analysis and Computer Vision: 1989." CAR-TR-483, CS-TR-2380, January 1990.

**ABSTRACT:** This paper presents a bibliography of nearly 1200 references related to computer vision and image analysis, arranged by subject matter. The topics covered include architectures; computational techniques; feature detection, segmentation, and image analysis; matching, stereo, and time-varying imagery; shape and pattern; color and texture; and three-dimensional scene analysis. A few references are also given on related topics, such as computational geometry, computer graphics, image input/output and coding, image processing, optical processing, visual perception, neural nets, pattern recognition, and artificial intelligence.

43. Jean-Michel Jolion, Peter Meer and Azriel Rosenfeld, "Generalized Minimum Volume Ellipsoid Clustering with Applications in Computer Vision." CAR-TR-485, CS-TR-2383, January 1990.

**ABSTRACT:** A novel clustering algorithm is proposed based on the minimum volume ellipsoid (MVE) robust estimator recently introduced in statistics. The MVE estimator identifies the least volume region containing  $h$  percent of the data points. The algorithm iteratively partitions the space into clusters without a priori information about their number. At each iteration the MVE estimator is applied several times with values of  $h$  decreasing from 0.5. A cluster is hypothesised for each ellipsoid. The shapes of these clusters are compared with shapes corresponding to a known unimodal distribution by the Kolmogorov-Smirnov test. The best fitting cluster is then removed from the space and a new iteration starts. Constrained random sampling keeps the amount of computation low. The clustering algorithm was successfully applied to several computer vision problems formulated in the feature space paradigm: decomposition of gray level and color image histograms and analysis of the Hough space.

44. Peter Meer, Doron Mintz and Azriel Rosenfeld, "Least Median of Squares Based Robust Analysis of Image Structure." CAR-TR-490, CS-TR-2428, March 1990.

**ABSTRACT:** Representation of an image by piecewise polynomial surfaces is an important computer vision task. Unfortunately the traditional least squares based techniques are prone to error when applied to nonhomogeneous regions. We introduce a new, robust algorithm which recovers the fit corresponding to the absolute majority (at least 51 percent) of the pixels in the processing window. The algorithm uses the least median of squares (LMedS) estimator recently introduced in the statistics literature. We show that the estimator may yield incorrect results when applied to images, and propose a two-stage procedure for the local description of the image structure and of the corrupting noise. Robust region growing can also be performed with the same technique. The goal of this paper is to introduce the LMedS paradigm to the computer vision community; the possible applications are not restricted to the ones treated here.

45. Saibal Banerjee, David Mount and Azriel Rosenfeld, "Pyramid Computation of Neighbor Distance Statistics in Dot Patterns." CAR-TR-491, CS-TR-2429, March 1990.

ABSTRACT: This paper describes an algorithm for computing statistics of Voronoi neighbor distances in a dot pattern, using a cellular pyramid computer, in a logarithmic number of computational steps. Given a set of dots in a square region of the digital plane, the algorithm determines with high probability the Voronoi neighbors of the dots in the interior of the region, and then computes statistics of the neighbor distances. An algorithm of this type may account for the ability of humans to perceive at a glance whether the dots in a pattern are randomly or regularly spaced, i.e. whether their neighbor distances have high or low variance.

46. Azriel Rosenfeld and Saibal Banerjee, "Maximum-Likelihood Edge Detection in Digital Signals." CAR-TR-492, CS-TR-2430, March 1990.

ABSTRACT: This paper treats the problem of edge detection in noisy piecewise-constant digital signals, using a maximum likelihood approach. Conventional edge detectors usually assume that the noise is Gaussian, and do not take advantage of prior knowledge about the ensemble of signals (aside from the assumption that the signals are piecewise constant). Our approach can handle noise that has an *arbitrary* probability density function; it also makes use of prior probability densities for the piece sizes and values in the signal ensemble.

47. Azriel Rosenfeld and T. Yung Kong, "Connectedness of a Set, its Complement, and their Common Boundary." CAR-TR-496, CS-TR-2435, March 1990.

ABSTRACT: We show that if a set  $S$  and its complement  $\bar{S}$  are both 8-connected, their common boundary  $B$  is connected, and conversely; and if  $S$  and  $\bar{S}$  are both 4-connected,  $B$  is a simple closed curve, and conversely. We also establish (with one exception) analogous results for 6-, 18-, and 26-connectedness in three dimensions.

48. Azriel Rosenfeld, "Fuzzy Rectangles." CAR-TR-499, CS-TR-2448, April 1990.

ABSTRACT: For a separable fuzzy subset of the plane, fuzzy connectedness, convexity, and orthoconvexity are all equivalent; a fuzzy set with these properties we call a *fuzzy rectangle*. We also define a *fuzzy convex polygon* in terms of an inf of "fuzzy halfplanes", and show that a fuzzy rectangle is a fuzzy convex polygon.

49. Jean-Michel Jolion, Peter Meer and Samira Bataouche, "Range Image Segmentation by Robust Clustering." CAR-TR-500, CS-TR-2456, April 1990.

**ABSTRACT:** A global, clustering-based method is proposed for segmentation of noisy range images. The locally estimated surface (facet) parameters are mapped into a feature space which is then decomposed into clusters representing large homogeneous regions in the range image. We make extensive use of recently introduced robust estimation techniques. M-estimators are employed for the recovery of the facet parameters, and minimum volume ellipsoid estimators for clustering. The quality of segmentation obtained in the presence of significant noise levels supports the use of the global technique.

50. Sven J. Dickinson, Alex P. Pentland and Azriel Rosenfeld, "Qualitative 3-D Shape Recovery using Distributed Aspect Matching." CAR-TR-505, CS-TR-2484, June 1990.

**ABSTRACT:** Recently, we proposed a modeling paradigm for 3-D object recognition integrating object-centered and viewer-centered models. Object models are object-centered constructions of 3-D volumetric primitives, offering an efficient indexing mechanism for large object databases. The 3-D primitives, in turn, are mapped into a set of viewer-centered aspects whose number depends only on the size of the set of primitives, not on the number of object models or on object model complexity. To minimize the size of the aspect set, we constrain the aspects to be invariant to minor changes in primitive shape, forcing the primitives to be qualitative in nature. To accommodate the representation of incomplete aspects arising from occluded model primitives, we introduce a hierarchical aspect representation based on aspect faces; the levels of the aspect hierarchy are linked together by a set of conditional probabilities derived from a statistical analysis of the aspects.

In this paper, we address the recovery of the 3-D primitives from an image. Given a set of regions in the image, we first label them as faces defined by the aspect hierarchy; if a face is occluded, it may be assigned multiple labels. To each face label, we assign a set of aspect labels as defined by the aspect hierarchy. We reduce the problem of recovering the 3-D primitives to the problem of partitioning the faces in the image into groups, each group representing an aspect of a primitive. We formulate the problem as a search through the possible aspect labelings of the image faces, and propose an effective heuristic based on the conditional probabilities linking together the levels of the aspect hierarchy. From a valid labeling of the image faces, we once again employ the aspect hierarchy to infer a set of 3-D primitives and extract their interconnectivity relations. The approach has been tested on several line drawing images.

51. Saibal Banerjee and Azriel Rosenfeld, "MAP Estimation of Piecewise Constant Digital Signals." CAR-TR-510, CS-TR-2512, July 1990.

**ABSTRACT:** Given a noisy signal  $g$ , if we have a probabilistic model for the ensemble of possible (non-noisy) signals  $f$  and a probabilistic model for the noise, we can in principle find the  $f$  that most likely gave rise to  $g$ , using Bayes' theorem; this most likely  $f$  is called the MAP ("maximum a posteriori") estimate of  $g$ . For discrete (digital) signals, the signal and noise probabilities can be arbitrary; it is not necessary to assume standard probability densities (e.g. Gaussian), which are unrealistic in many situations. However, MAP estimation is not commonly used even for digital signals, because the size of the set of possible  $f$ s is often exponential. In this paper we show, for piecewise constant  $f$ s, how the most likely  $f$  can be found using dynamic programming techniques that require only polynomial space and time. We also illustrate the performance of MAP estimation in recovering parameters of a simple signal or ensemble of signals in the presence of various types of noise.

52. Rae-Hong Park and Peter Meer, "Multiresolution Adaptive Least Squares Smoothing of Images." CAR-TR-513, CS-TR-2527, August 1990.

**ABSTRACT:** A non-iterative, parameter-free image smoothing method is presented which handles both additive and multiplicative noise. First for every pixel the largest centered neighborhood ( $7 \times 7$ ,  $5 \times 5$  or  $3 \times 3$ ) containing no discontinuity is sought. The selection is made by comparing a local discontinuity measure with its robust global estimate corresponding to homogeneous neighborhoods. If no discontinuity is present, the pixel is assigned the spatial mean computed in the neighborhood. Around discontinuities the adaptive least squares smoothing method of Kuan *et al.* is applied in  $3 \times 3$  neighborhoods. The performance of the multiresolution algorithm is compared with flat-facet smoothing and adaptive smoothing for additive noise. The smoothing of synthetic aperture radar images is used as an example for multiplicative noise. The proposed algorithm belongs to the more general class of consensus vision methods.

53. Allen C. Sher and Azriel Rosenfeld, "Pyramid Cluster Detection and Delineation by Consensus." CAR-TR-519, CS-TR-2555, October 1990.

**ABSTRACT:** Pyramid algorithms for compact region detection and delineation can be used to detect compact dot clusters on a sparsely dotted background. When the delineation process is applied to a detected cluster, it yields "ragged" results which are sensitive to the position of the cluster in the image. But if the process is applied to a set of shifted versions of the cluster, the results can be combined into an acceptable delineation. This illustrates a general "consensus" approach to combining multiple techniques, or multiple versions of the same technique, to obtain more reliable results.

54. Prabir Bhattacharya and Azriel Rosenfeld, "Polygonal Arcs and Polygons in Three Dimensions." CAR-TR-520, CS-TR-2556, October 1990.

ABSTRACT: Determining whether a nonselfintersecting closed curve in three dimensions is knotted, or whether two such curves are linked, can be done by computing the curves' homotopy groups. Generators and relations that define this group for a given curve can be determined by examining the parallel projection of the curve onto a plane, provided the projection is nondegenerate in a certain sense (a "Wirtinger projection") and provided we know, whenever the curve crosses itself in the projection, which of the two branches is closer to the viewer.

We show in this paper that when the curve is a polygon, most parallel projections are Wirtinger projections. We also show that a polygon or polygonal arc can be reconstructed from three Wirtinger projections in noncoplanar directions, even without the information about the crossings. We review the basics of knot theory (and provide background material about groups and topological spaces in appendices), and give conditions for a "knot" to be trivial (i.e., unknotted) and for two trivial knots to be (un)linked. Finally, we consider polygonal arcs and polygons that are isothetic (i.e., all their sides are parallel to the coordinate axes); we show how to represent a polygon(al arc), up to translation and rotation, by a sequence of side lengths and two-bit numbers representing  $90^\circ$  rotations, and how to derive its properties from this representation.

55. Sankar K. Pal and Azriel Rosenfeld, "A Fuzzy Medial Axis Transformation Based on Fuzzy Disks." CAR-TR-524, CS-TR-2560, November 1990.

ABSTRACT: A fuzzy disk with center  $P$  is a fuzzy set in which membership depends only on distance from  $P$ . For any fuzzy set  $f$ , there is a maximal fuzzy disk  $g_P^f \leq f$  centered at every point  $P$ , and  $f$  is the sup of the  $g_P^f$ 's. (Moreover, if  $f$  is fuzzy convex, so is every  $g_P^f$ , but not conversely.) We call a set  $S_f$  of points  $f$ -sufficient if every  $g_P^f \leq f_Q^f$  for some  $Q$  in  $S_f$ ; evidently  $f$  is then the sup of the  $g_Q^f$ 's. In particular, in a digital image, the set of  $Q$ 's at which  $g^f$  is a (nonstrict) local maximum is  $f$ -sufficient. This set is called the *fuzzy medial axis* of  $f$ , and the set of  $g_Q^f$ 's is called the fuzzy medial axis transformation (FMAT) of  $f$ . These definitions evidently reduce to the standard ones if  $f$  is a crisp set. Unfortunately, for an arbitrary  $f$ , specifying the FMAT may require more storage space than specifying  $f$  itself.

56. Doron Mintz, Peter Meer and Azriel Rosenfeld, "Consensus by Decomposition: A Paradigm for Fast High Breakdown Point Robust Estimation." CAR-TR-525, CS-TR-2571, December 1990.

ABSTRACT: Robust high breakdown point techniques are gaining increasing popularity in computer vision since they can tolerate up to half the data being severely corrupted. The least median of squares (LMedS) estimator is the best known example of such techniques. We show that the attractive properties of the LMedS estimator do not hold when all the data is corrupted by zero mean noise. We propose a new "consensus by decomposition" (CBD) algorithm which

preserves the properties of LMedS up to low signal-to-noise ratios while achieving a significant speed-up relative to LMedS. The CBD estimator uses a different paradigm than LMedS. The data is decomposed in both the spatial and parameter domains. A separate distribution is built for every parameter and the distribution is analyzed with a new, enhanced mode detection procedure. The superiority of the CBD estimator is proved by extensive simulations.

57. Rae-Hong Park and Peter Meer, "Edge-Preserving Artifact-Free Smoothing with Image Pyramids." CAR-TR-527, CS-TR-2576, December 1990.

**ABSTRACT:** We present a hierarchical implementation of an edge-preserving smoothing algorithm on the  $2 \times 2$  pyramid structure. The smoothed pixel values are chosen from the first three levels of the pyramid and the original image. The reduced resolution representations are analyzed in a top-down fashion by comparing the local variances with the corresponding global noise variance estimates. The global estimates are also computed in the pyramid. Close to edges the pixel values are obtained by adaptive least squares. The artifacts of region-based smoothing are eliminated by pixelwise averaging over a set of outputs obtained with the input image shifted within the  $8 \times 8$  block of the level-three parent.

58. Azriel Rosenfeld, "Image Analysis and Computer Vision: 1990." CAR-TR-529, CS-TR-2578, January 1991.

**ABSTRACT:** This paper presents a bibliography of over 1600 references related to computer vision and image analysis, arranged by subject matter. The topics covered include architectures; computational techniques; feature detection, segmentation, and image analysis; matching, stereo, and time-varying imagery; shape and pattern; color and texture; and three-dimensional scene analysis. A few references are also given on related topics, such as computational geometry, computer graphics, image input/output and coding, image processing, optical processing, visual perception, neural nets, pattern recognition, and artificial intelligence.